NAME

cubocteversion − Displays a cuboctahedron eversion.

SYNOPSIS

cubocteversion [−−display *host:display.screen*] [−−install] [−−visual *visual*] [−−window] [−−root] [−−window−id *number*] [−−delay *usecs*] [−−fps] [−−eversion-method *method*] [−−morin-denner] [−−apery] [−−mode *display-mode*] [−−surface] [−−transparent] [−−edges *edge-mode*] [−−self-intersections *self-intersection-mode*] [−−colors *color-scheme*] [−−twosided-colors] [−−face-colors] [−−earth-colors] [−−deformation-speed *float*] [−−projection *projection-mode*] [−−perspective] [−−orthographic] [−−transparency *transparency-method*] [−−correct-transparency] [−−approximate-transparency] [−−standard-transparency] [−−speed-x *float*] [−−speed-y *float*] [−−speed-z *float*]

DESCRIPTION

The *cubocteversion* program shows a cuboctahedron eversion, i.e., a smooth deformation (homotopy) that turns a cuboctahedron inside out. During the eversion, the deformed cuboctahedron is allowed to intersect itself transversally. However, no fold edges or non-injective neighborhoods of vertices are allowed to occur.

The cuboctahedron can be deformed with two eversion methods: Morin-Denner or Apéry. The Morin-Denner cuboctahedron eversion method is described in the following two papers: Richard Denner: "Versions polyédriques du retournement de la sphère", L'Ouvert 94:32-45, March 1999; Richard Denner: "Versions polyédriques du retournement de la sphère, retournement du cuboctaèdre", L'Ouvert 95:15-36, June 1999. The Apéry cuboctahedron eversion method is described in the following paper: François Apéry: "Le retournement du cuboctaèdre", Prépublication de l'institut de recherche mathématique avancée, Université Louis Pasteur et C.N.R.S., Strasbourg, 1994.

The deformed cuboctahedron can be projected to the screen either perspectively or orthographically.

There are three display modes for the cuboctahedron: solid, transparent, or random. If random mode is selected, the mode is changed each time an eversion has been completed.

The edges of the faces of the cuboctahedron can be visualized in three modes: without edge tubes, with edge tubes, or random. If edge tubes are selected, solid gray tubes are displayed around the edges of the cuboctahedron. This makes them more prominent. If random mode is selected, the mode is changed each time an eversion has been completed.

During the eversion, the cuboctahedron must intersect itself. It can be selected how these self-intersections are displayed: without self-intersection tubes, with self-intersection tubes, or random. If self-intersection tubes are selected, solid orange tubes are displayed around the self-intersections of the cuboctahedron. This makes them more prominent. If random mode is selected, the mode is changed each time an eversion has been completed.

The colors with with the cuboctahedron is drawn can be set to two-sided, face, earth, or random. In two-sided mode, the cuboctahedron is drawn with magenta on one side and cyan on the other side. In face mode, the cuboctahedron is displayed with different colors for each face. The colors of the faces are identical on the inside and outside of the cuboctahedron. Colors on the northern hemi-cuboctahedron are brighter than those on the southern hemi-cuboctahedron. In earth mode, the cuboctahedron is drawn with a texture of earth by day on one side and with a texture of earth by night on the other side. Initially, the earth by day is on the outside and the earth by night on the inside. After the first eversion, the earth by night will be on the outside. All points of the earth on the inside and outside are at the same positions on the cuboctahedron. Since an eversion transforms the cuboctahedron into its inverse, the earth by night will appear with all continents mirror reversed. If random mode is selected, the color scheme is changed each time an eversion has been completed.

It is possible to rotate the cuboctahedron while it is deforming. The rotation speed for each of the three coordinate axes around which the cuboctahedron rotates can be chosen arbitrarily.

BRIEF DESCRIPTION OF THE CUBOCTAHEDRON EVERSION BASICS

A sphere eversion turns the standard embedding of the unit sphere inside-out in a smooth manner. Creases, pinch points, holes, etc. may not occur during the eversion. However, the sphere may intersect itself during

the eversion. In mathematical terms, the eversion is a regular homotopy between the sphere and the sphere point reflected at its center. A convex bounded polyhedron of Euler characteristic 2 is homeomorphic to a sphere. Since a polyhedron does not have a continuous tangent bundle, it cannot be everted by a regular homotopy, which requires the tangent bundle induced by the homotopy to be continuous. Instead, it is required that polyhedron does not develop fold edges during the eversion and that a neighborhood of each vertex is injective throughout the eversion. Fold edges occur whenever two faces that share an edge become coplanar and all vertices of the two faces lie on the same side of the edge in the plane in which they are coplanar. Furthermore, it is required that all self-intersections between edges that occur during the eversion are transversal, which means that they must not occur at the vertices of the edges.

Any eversion of the sphere (smooth or polyhedral) must contain a quadruple point. This is a point in which four different parts of the deformed sphere intersect transversally. For a polyhedron, this means that four different faces must intersect transversally. Four faces are defined by four planes, each of which, in turn, is defined by three vertices. By the above requirements, none of the twelve vertices that define the four planes may coincide. Therefore, the minimum number of vertices of a polyhedron that allows it to be everted is twelve. The cuboctahedron has twelve vertices and the papers cited above show that a cuboctahedron can indeed be everted.

A cuboctahedron has 14 faces: six squares and eight equilateral triangles. To perform the eversion, the cuboctahedron is oriented such that two opposite squares are horizontal. One of these squares corresponds to the north polar region and one to the south polar region if the cubctahedron is identified with the round sphere. The four remaining squares are vertical and lie in the tropical region around the equator. Each square is then divided into two isosceles right triangles. The four tropical squares are divided along the equator and the north and south pole squares are divided in orthogonal directions: the edge introduced in the north pole square is orthogonal to that introduced in the south pole square. This results in a triangulated version of the cuboctahedron with 12 vertices, 30 edges, and 20 triangular faces. This is the version of the cuboctahedron that can be everted.

BRIEF DESCRIPTION OF THE MORIN-DENNER CUBOCTAHEDRON EVERSION METHOD

The approach of Morin and Denner is to evert the cuboctahedron in 44 steps, resulting in 45 different polyhedra that occur as models. The eversion is symmetric in time, so the 44 steps can be visualized by time running from -22 to 22. Of the 45 models, 44 possess a twofold rotational symmetry. The halfway model at time 0 possesses a fourfold rotational symmetry. The halfway model is the model at which the cuboctahedron is turned halfway inside-out. In each of the 44 steps, two vertices of the cuboctahedron are moved along two respective straight lines, each of which is an edge or an extension of an edge of the cuboctahedron. After the eversion has been completed, the inside of the cuboctahedron lies on the outside. Furthermore, all points of the everted cuboctahedron lie at the antipodal points of the original cuboctahedron.

The following description assumes that the cuboctahedron is visualized in two-sided color mode. In the first 16 steps, the magenta cuboctahedron is deformed into a magenta polyhedron that Morin and Denner call the bicorne. During this phase, no self-intersections occur. Topologically, the bicorne is still an embedded sphere. The next twelve steps, from time -6 to 6, are the most interesting steps of the eversion: the cuboctahedron intersects itself. It no longer is an embedding but an immersion. In this phase, progressively more of the cyan inside becomes visible. These steps are shown at a two times slower speed compared to the rest of the steps. At time 6, the eversion has produced a cyan bicorne. At this step, the cuboctahetron has been everted: it is an embedding of the everted sphere. In the remaining 16 steps, the cyan bicorne is deformed to the everted cuboctahedron.

BRIEF DESCRIPTION OF THE APÉRY CUBOCTAHEDRON EVERSION METHOD

The original approach of Apéry is to evert the cuboctahedron in four steps, resulting in five different polyhedra that occur as models. The eversion is symmetric in time, so the four steps can be visualized by time running from -2 to 2. Of the five models, four possess a twofold rotational symmetry. The halfway model at time 0 possesses a fourfold rotational symmetry. The halfway model is the model at which the cuboctahedron is turned halfway inside-out. In addition to the start and end models at times -2 and 2, which both are cuboctahedra, and the halfway model at time 0, the two intermediate models at times -1 and 1 are embeddings of the cuboctahedron. Apéry calls them gastrula because they correspond to a cuboctahedron in which the northern hemi-cuboctahedron has been pushed downwards so that it lies inside the southern

hemi-cuboctahedron. In each of the four steps, the cuboctahedron is deformed by linearly interpolating the corresponding vertices between two successive models. After the eversion has been completed, the inside of the cuboctahedron lies on the outside. Furthermore, all points of the everted cuboctahedron lie at the antipodal points of the original cuboctahedron.

During the development of this program, it was discovered that the linear interpolation between the cuboctahedron and the gastrula causes the deformed cuboctahedron to intersect itself for a brief period of time shortly before the gastrula is reached. Therefore, an additional model, devised by François Apéry and called pre-gastrula by him, was inserted at times -1.25 and 1.25. This additional model avoids the self-intersections before the gastrula is reached. The rest of Apéry's approach remains unaffected: the vertices are interpolated linearly between successive models.

The following description assumes that the cuboctahedron is visualized in two-sided color mode. In the first two steps, the magenta cuboctahedron is deformed into a magenta gastrula. During this phase, no self-intersections occur. Topologically, the gastrula is still an embedded sphere. The next two steps, from time -1 to 1, are the most interesting steps of the eversion: the cuboctahedron intersects itself. It no longer is an embedding but an immersion. In this phase, progressively more of the cyan inside becomes visible. At time 1, the eversion has produced a cyan gastrula. At this step, the cuboctahetron has been everted: it is an embedding of the everted sphere. In the remaining two steps, the cyan gastrula is deformed to the everted cuboctahedron.

OPTIONS

cubocteversion accepts the following options:

−−window
Draw on a newly-created window. This is the default.

−−root Draw on the root window.

−−window−id *number*
Draw on the specified window.

−−install
Install a private colormap for the window.

−−visual *visual*
Specify which visual to use. Legal values are the name of a visual class, or the id number (decimal or hex) of a specific visual.

−−delay *microseconds*
How much of a delay should be introduced between steps of the animation. Default 20000, or 1/50th second.

−−fps Display the current frame rate, CPU load, and polygon count.

The following three options are mutually exclusive. They determine which cuboctahedron eversion method is used.

−−eversion-method random
Use a random cuboctahedron eversion method (default).

−−eversion-method morin-denner (Shortcut: **−−morin-denner**)
Use the Morin-Denner cuboctahedron eversion method.

−−eversion-method apery (Shortcut: **−−apery**)
Use the Apéry cuboctahedron eversion method.

The following three options are mutually exclusive. They determine how the deformed cuboctahedron is displayed.

−−mode random
Display the cuboctahedron in a random display mode (default).

−−mode surface (Shortcut: **−−surface**)
Display the cuboctahedron as a solid surface.

−−mode transparent (Shortcut: **−−transparent**)
Display the cuboctahedron as a transparent surface.

The following three options are mutually exclusive. They determine whether the edges of the cuboctahedron are displayed as solid gray tubes.

−−edges random
Randomly choose whether to display edge tubes (default).

−−edges on
Display the cuboctahedron with edge tubes.

−−edges off
Display the cuboctahedron without edge tubes.

The following three options are mutually exclusive. They determine whether the self-intersections of the deformed cuboctahedron are displayed as solid orange tubes.

−−self-intersections random
Randomly choose whether to display self-intersection tubes (default).

−−self-intersections on
Display the cuboctahedron with self-intersection tubes.

−−self-intersections off
Display the cuboctahedron without self-intersection tubes.

The following four options are mutually exclusive. They determine how to color the deformed cuboctahedron.

−−colors random
Display the cuboctahedron with a random color scheme (default).

−−colors twosided (Shortcut: **−−twosided-colors**)
Display the cuboctahedron with two colors: magenta on one side and cyan on the other side.

−−colors face (Shortcut: **−−face-colors**)
Display the cuboctahedron with different colors for each face. The colors of the faces are identical on the inside and outside of the cuboctahedron. Colors on the northern hemi-cuboctahedron are brighter than those on the southern hemi-cuboctahedron.

−−colors earth (Shortcut: **−−earth-colors**)
Display the cuboctahedron with a texture of earth by day on one side and with a texture of earth by night on the other side. Initially, the earth by day is on the outside and the earth by night on the inside. After the first eversion, the earth by night will be on the outside. All points of the earth on the inside and outside are at the same positions on the cuboctahedron. Since an eversion transforms the cuboctahedron into its inverse, the earth by night will appear with all continents mirror reversed.

The following option determines the deformation speed.

−−deformation-speed *float*
The deformation speed is measured in percent of some sensible maximum speed (default: 20.0).

The following three options are mutually exclusive. They determine how the deformed cuboctahedron is projected from 3d to 2d (i.e., to the screen).

−−projection random
Project the cuboctahedron from 3d to 2d using a random projection mode (default).

−−projection perspective (Shortcut: **−−perspective**)
Project the cuboctahedron from 3d to 2d using a perspective projection.

−−projection orthographic (Shortcut: **−−orthographic**)
Project the cuboctahedron from 3d to 2d using an orthographic projection.

The following three options are mutually exclusive. They determine which transparency algorithm is used to display the transparent faces of the cuboctahedron. If correct transparency is selected, a correct but slower algorithm is used to render the transparent faces. If the frame rate of this algorithm is too slow and results in a jerky animation, it can be set to one of the other two modes. If approximate transparency is selected, an transparency algorithm that provides an approximation to the correct transparency is used. Finally, if standard transparency is selected, a transparency algorithm that only uses standard OpenGL transparency rendering features is used. It results in a lower-quality rendering of the transparent faces in which the appearance depends on the order in which the faces are drawn. The approximate and standard transparency algorithms are equally fast and, depending on the GPU, can be significantly faster than the correct transparency algorithm. The correct and approximate transparency algorithms are automatically switched off if the OpenGL version supported by the operating system does not support them (for example, on iOS and iPadOS).

−−transparency correct (Shortcut: **−−correct-transparency**)
Use a transparency algorithm that results in a correct rendering of transparent surfaces (default).

−−transparency approximate (Shortcut: **−−approximate-transparency**)
Use a transparency algorithm that results in an approximately correct rendering of transparent surfaces.

−−transparency standard (Shortcut: **−−standard-transparency**)
Use a transparency algorithm that uses only standard OpenGL features for the rendering of transparent surfaces.

The following three options determine the rotation speed of the deformed cuboctahedron around the three possible axes. The rotation speed is measured in degrees per frame. The speeds should be set to relatively small values, e.g., less than 4 in magnitude.

−−speed-x *float*
Rotation speed around the x axis (default: 0.0).

−−speed-y *float*
Rotation speed around the y axis (default: 0.0).

−−speed-z *float*
Rotation speed around the z axis (default: 0.0).

INTERACTION

If you run this program in standalone mode, you can rotate the deformed cuboctahedron by dragging the mouse while pressing the left mouse button. This rotates the cuboctahedron in 3d. To examine the deformed cuboctahedron at your leisure, it is best to set all speeds to 0. Otherwise, the deformed cuboctahedron will rotate while the left mouse button is not pressed.

ENVIRONMENT

DISPLAY
to get the default host and display number.

XENVIRONMENT
to get the name of a resource file that overrides the global resources stored in the RESOURCE_MANAGER property.

XSCREENSAVER_WINDOW
The window ID to use with *−−root*.

SEE ALSO

X(1), **xscreensaver**(1),

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